

## User Notes for the “Diameters” Program

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### Introduction

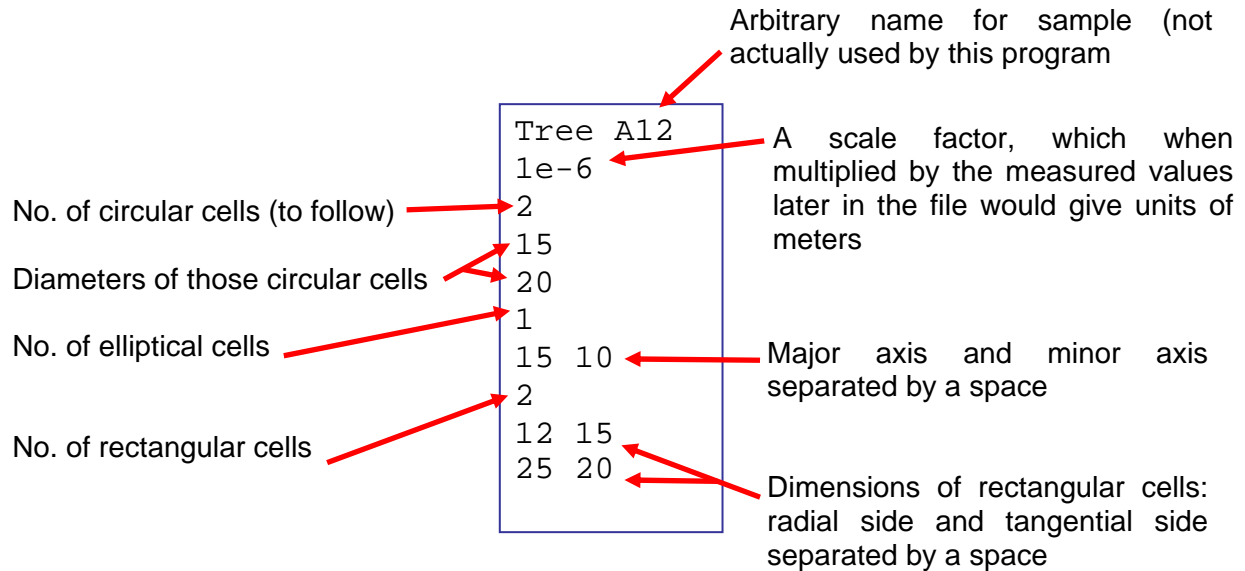
Studies of water flow through the xylem of plants may involve measurements of the diameters of the tracheids or vessels as conduits. A useful statistic might be the average diameter of these cells. But their cross-sections are often not circular. Fluid mechanics provides us with a number of formulae describing flow through conduits with non-circular cross-sections (see Appendix) and so one approach would be to convert all cells to circular conduits of equivalent conducting ability. To this end, a program was developed that would read through data files containing a list of conduit measurements, perform the necessary circularization, and calculate a few simple statistics.

### Running the program

Upon startup, the program creates a window like that shown here. The first step is likely to be selecting a data file for input. The expected format for this file may be seen by selecting “Help...File format...” from the program’s menu system.



Here is a sample data file:



The length of the file will vary of course depending on the number of cells measured - the number of cells for each shape is entered before that group of cells. The program does a little bit of input file error checking, such that if the numbers of rows of cell data is less than the numbers of cells specified, an error message will be presented.

The idea with rectangular cells (unlike for the elliptical cells) was to maintain some info about the orientation of those cells. So their dimensions can be entered as the cell width along the radial face and tangential face because most of the rectangular cells that I have seen have their walls oriented this way. The program will keep track of both radial & tangential dimensions as well as long side & short side dimensions to give a sense (on average) if the cells have their long or short sides in the radial or tangential orientation. Such considerations of orientation do not affect the circularization calculations, so you can always enter data for rectangular conduits in any order and ignore the output referring to radial and tangential orientations.

Data files are selected with the "File...Open..." menu item. The program uses a standard MS Windows file dialog, where you can select files for input. The program is expecting data files to have a "dat" extension to the filename.

The program was designed to allow for combining multiple data files using two possible approaches. First, the file dialog is one that allows for selecting multiple files in any of the usual Windows ways. Second, the file open process may be repeated many times and data from files will be accumulated. This means that when moving on to new data files where you do not want to keep combining them, you must select "File...Clear" to erase all the old data in the program.

After data files have been read, the program window will display all the statistics. Also, the second row in the window shows the number of files that have been accumulated.

Remember that when one wants to start over with new data (except when the program is first started before any files have been read), first select “File...Clear” to purge all of the current data in the program.

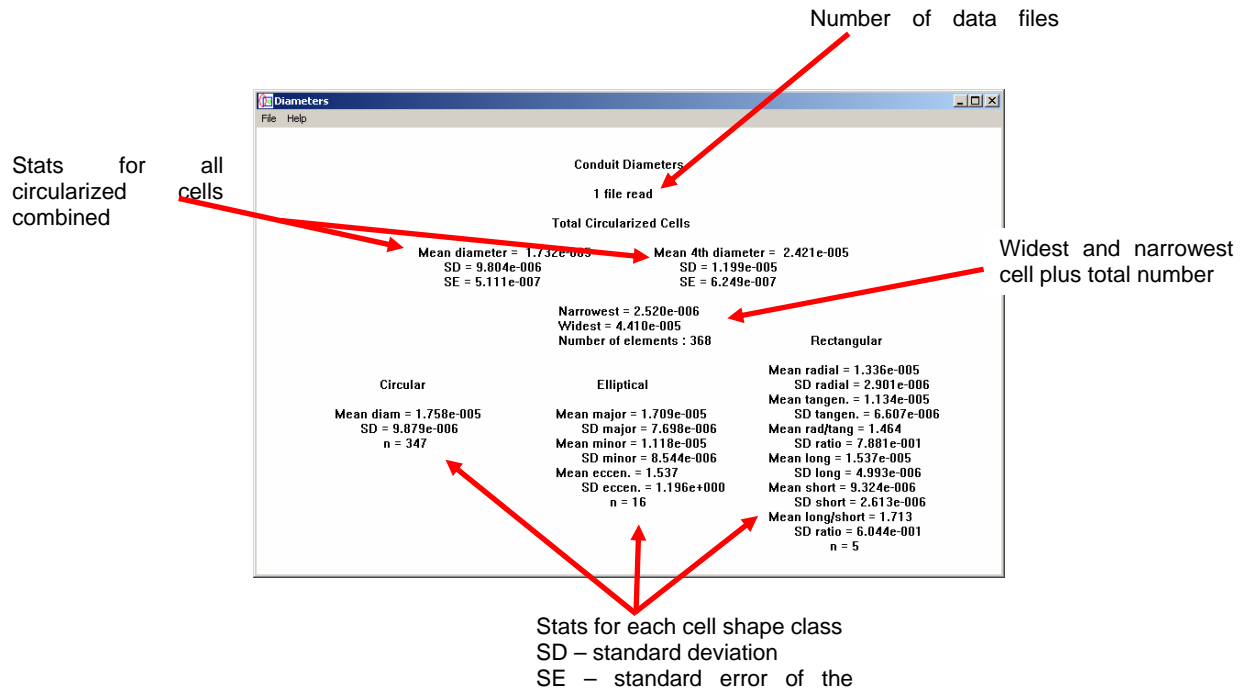
### Program output

Here’s a typical program window - in this case after reading a sample data file called “sycamore.dat”.

Mean 4<sup>th</sup> diameter uses a formula like:

$$d^* = \sqrt[4]{\frac{\sum_{i=1}^N d_i^4}{N}}$$

that some users may feel is a sensible approach because for simple, ideal pipes, conducting ability is related to the fourth power of diameter. In this manner, the mean is “weighted” more heavily towards wider cells because they presumably contribute more to flow than narrow cells.



For elliptical and rectangular groups, some additional data are provided on cell shape and orientation (eccentricity for elliptical cells). For rectangular cells, radial and tangential widths and their ratio along with the long and short side widths and their ratio is also given. These ratios usual differ, but if for example every cell had its radial side as the short side, then the ratios would be the same.

All distance are given in meter units assuming the correct scale factors were provided in the data files. Actually, the formulae used in program calculations (see Appendix) should be dimensionally consistent, and so in principle one could enter data in, say, micrometer units, use 1 for the scale factor and then the program will show output in micrometer units. This was tested briefly and seems to work.

## Appendix

### Calculation of equivalent circular conduits from elliptical and rectangle conduits

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Tracheids of Douglas-fir observed in cross-section were rarely circular, with elliptical and rectangular cells occurring in roughly equal numbers. A common basis for comparing tracheid sizes was needed and because water transport is one of the primary functions of these tracheids, conducting ability was chosen as a basis. The hydraulic conductance ( $K$ ) of simple conduits is nonlinearly related to the size of the conduit and is given for these shapes as (see Langlois, 1964 and White, 1991):

$$K^{circular} = \frac{\pi d^4}{128\eta} \quad (1)$$

where  $d$  is the conduit diameter and  $\eta$  is the fluid viscosity;

$$K^{elliptical} = \frac{\pi}{4\eta} \frac{(ab)^3}{(a^2 + b^2)} \quad (2)$$

where  $a$  is the semi-major axis and  $b$  is the semi-minor axis;

$$K^{rectangular} = \frac{4ba^3}{3\eta} \left( 1 - \frac{192a}{\pi^5 b} \sum_{i=1,3,5}^{\infty} \frac{\tanh(i\pi b/2a)}{i^5} \right) \quad (3)$$

where, here,  $a$  is the semi-long side and  $b$  is the semi-short side of the rectangle. Equivalent diameters for elliptical conduits can be calculated by equating equations 1 and 2, and for rectangular conduits by equating equations 1 and 3, giving:

$$d = \left( \frac{32(ab)^3}{(a^2 + b^2)} \right)^{1/4} \quad (\text{for ellipses}) \quad (4)$$

$$d = \left[ \frac{512ba^3}{3\pi} \left( 1 - \frac{192a}{\pi^5 b} \sum_{i=1,3,5}^{\infty} \frac{\tanh(i\pi b/2a)}{i^5} \right) \right]^{1/4} \quad (\text{for rectangles}) \quad (5)$$

Fortunately, the series summation in Equation (5) converges such that 32 terms ( $i$  up to 65) gives a sum accurate to at least 7 significant figures for rectangular conduits with

sides ranging from 1:1 to 10:1. A program was developed (using Microsoft Visual C++ for Windows) to carry out these calculations along with statistical data such as mean cell diameter and standard deviation.

#### Literature Cited

Langlois, WE. 1964. Slow Viscous Flow. Macmillan, New York.

White, FM. 1991. Viscous Fluid Flow. McGraw-Hill, New York.